Aleksei A. Abrikosov

1928–2017

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
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ALEKSEI ALEKSEEVICH **ABRIKOSOV**

June 25, 1928–March 29, 2017

Elected to the NAS, 2000

By M. R. Norman

Shortly after the 2003 announcement that Aleksei Abrikosov had won the Nobel Prize in Physics, a number of colleagues took Alex to lunch at a nearby Italian restaurant. During lunch, one of the Russian visitors exclaimed that Alex should get a second Nobel Prize, this time in Literature for his famous "AGD" book with Lev Gor'kov and Igor Dzyaloshinskii (Methods of Quantum Field Theory in Statistical Physics.) Somewhat taken aback, I looked closely at this individual and realized that he was deadly serious. Although I could imagine the reaction of the Nobel Literature committee to such a book (for a lay person, perhaps analogous to trying to read Finnegan's Wake), I had to admit that my own copy of this book is quite dog-eared, having been put to good use over the years. In fact, you know you have made it in physics when your book gets a Dover edition. One of the most charming pictures I ever saw was a rare drawing in color that Alexei Tsvelik did (commissioned by Andrei Varlamov for Alex's 50th birthday) that was proudly displayed in Alex's home in Lemont, IL. It showed Alex with his fingers raised in a curled fashion as in the habit of medieval Popes. In his hand is an object resembling the Bible, but with the initials "AGD" on its front. Around this portrait are drawings of four of Alex's students, presumably learning at the feet of the Master.

But AGD is not the work Alex is best known for. In the early 1950s, he was trying to reconcile experimental data of his colleague Nikolay Zavaritskii with the recently published phenomenological theory of Ginzburg and Landau on superconductivity. He realized (1952) that Zavaritskii's data concerning the critical magnetic field at which superconductivity disappeared in thin films could be explained if he considered a case that Ginzburg and Landau had not considered, one where the surface energy between the normal and superconducting regions was negative. Alex referred to these as superconductors of the second group (now called type II superconductors). Afterwards, Alex realized that there were two critical fields, a lower one where magnetic flux first pene-

A drawing by Alexei Tsvelik that was commissioned by Andrey Varlamov for Alex's 50th birthday (1978). Note the copy of AGD Alex is holding. The students surrounding him are (from left to right) Alex Kovner, Ivan Ryzhkin, Andrey Varlamov, and Sergei Mukhin.

trated the sample, and a much higher one where superconductivity was destroyed. In between these fields, he predicted the existence of a lattice of quantized vortices. This discovery, one of the few where an analytic many-body wave function was derived, helped resolve puzzling data going back to the work of de Haas, Lev Shubnikov and others in the 1930s. This concept was so radical that Lev Landau initially dismissed it. But upon Richard Feynman's elucidation of vortices in liquid helium, Landau relented and allowed Alex to publish his paper (1957). As with many great papers, it was largely ignored in the beginning, but the discovery of the vortex lattice by neutron scattering along with its direct visualization by decoration experiments (sprinkling iron filings on the surface) in the 1960s brought the paper to the forefront. As Alex pointed out in his Nobel lecture, it is actually type I superconductors which are exotic, as the vast majority of important superconductors are type II. Perhaps most ubiquitously known for their role in MRI machines, type II superconductors power the Large Hadron Collider that was used to discover the Higgs boson, a concept which, itself, originated from superconductivity.

Alex was born in Moscow on June 25, 1928. He came from a well-known Russian family who owned a famous chocolate company (a poster of which you can find on Amazon), whose customers included the Czar's family. Alex's uncle, Dmitrii, was the last envoy of the Russian Empire in Japan, and his memoirs are available in English. Alex didn't even know of his uncle's existence until he was an older adult (but the KGB did, which was the reason Alex was not allowed to participate in the Soviet nuclear weapons program.) Alex's father was a world-famous pathologist, perhaps best known for leading the team that embalmed Lenin's body for display in the Lenin Mausoleum (he also did the autopsy for Lenin, a controversy given, claims, never proven, that he doctored the reports under orders to hide the fact that Lenin had syphilis). He was Vice President of the Academy of Medical Sciences and won a number of major awards (Stalin Prize, Hero of Socialist Labor, etc.). Alex's mother, Fanny, a former student of Alex's father, was a well-known pathologist in her own right.

Alex graduated from High School in 1943 during the midst of the German invasion, and in 1945 he transferred to the Physics Department at Moscow State University where he graduated three years later with an M.Sc. degree. He was then accepted to graduate school at the Institute for Physical Problems (later known as the Kapitza Institute), with his advisor being the future Nobel Laureate, Lev Landau. Alex passed the famous Landau minimum at the age of 19, and then obtained his PhD in 1951 with a thesis on thermal diffusion in plasmas. Landau wanted to keep Alex on at the institute, but there was a snag. A KGB official looked over Alex's file and noticed that not only was his mother Jewish, but her name was such as to make the KGB official suspect that Alex was Landau's nephew. Then fate intervened. The leader of Mongolia, Marshal Choibalsan, died while on a visit to Moscow, and Alex's mother performed the autopsy. When the KGB official found out about this, he was impressed enough to relent and allowed Alex to stay with Landau. The rest is history, so to speak.

The work for which Alex received his doctorate (equivalent to a habilitation) in 1955 was not his famous type II superconductivity work, but rather equally interesting work on quantum electrodynamics. This work was facilitated by Alex and Isaak Khalatnikov learning techniques developed in the West by Feynman and others, with important guidance coming from Landau about which diagrams to sum. Initially, they made a sign error in this work, and Landau realized they had discovered asymptotic freedom. When the sign error was pointed out to them (in one story, by Galanin and Tamm, in another by Galanin and Ioffe), they realized they had instead identified the so-called "Moscow zero," which can be interpreted as the renormalized charge vanishing in the point interaction limit due to complete screening. Asymptotic freedom was later discovered in the context of quantum chromodynamics by Gross, Politzer, and Wilczek, resulting in a

Nobel prize for them in 2004 (a fact that Isaak Khalatnikov ruefully acknowledged when I met him at the Landau Institute in Chernogolovka in 2007).

A famous picture of Landau's group in Moscow (1956). Sitting: Lyudmila Prozorova, Alex, Isaak Khalatnikov, Lev Landau, Evgeny Lifshitz. Standing: Semyon Gershtein, Lev Pitaevskii, Leonid Vainshtein, Robert Arkhipov, Igor Dzyaloshinskii.

During the same period as the quantum electrodynamics work, Alex did his Nobel-Prize-winning work on type II superconductivity. As in most truly profound work, the implications go far beyond what was originally intended. First, vortices are topological entities (the phase of the superconducting order parameter winds by 2π around the vortex core, i.e., the zero of the superconducting wave function). This led to consideration of such objects in other contexts, particularly in high energy physics. Perhaps the best-known example, though, is again in the field of superconductivity. By the Mermin-Wagner-Coleman theorem, a finite temperature XY phase transition (which superconductivity is equivalent to, since its order parameter is a complex scalar) is not allowed in two dimensions because of thermal fluctuations. It was later shown that a topological transition (binding/unbinding of vortex pairs) does occur, the famous Berezinskii-Kosterlitz-Thouless transition (for which the latter two won the Nobel Prize in 2016). To this can be added the prevalence of vortex-based ideas in microscopic theories of the fractional quantum Hall effect and quantum spin liquids, and the recent discovery of electric (as opposed to magnetic) vortices in certain transition metal oxides.

Besides the above, Alex also worked on the possibility of metallic hydrogen in the core of gas giants (much in vogue these days given recent high-pressure experiments claiming to have achieved the metallic phase), and then at the end of the 1950s, he turned to the microscopic theory of superconductivity after learning about the work of Bogoliubov. He and Khalatnikov generalized Bogoliubov's results to finite temperature, but their two papers were rejected by Evgeny Lifshitz because by then, the Russians had become aware of the papers by Bardeen, Cooper and Schrieffer (Nobel Prize, 1972) where these results were derived in a different manner (their papers, though, were later published after Valatin had independently derived some of their results). By then, Alex became interested in applying some of the techniques he had learned from the work in quantum electrodynamics to the problem of superconductivity. In a series of famous papers, he and Lev Gor'kov exploited an obscure formalism by Matsubara to extend zero temperature quantum field theory to finite temperatures. Their crucial insight was the realization that one could state the problem as a Fourier sum, with the inverse of the temperature playing the role of imaginary time. Since much was known analytically about such sums, hard problems, which would have previously involved complicated frequency integrals, became much easier to solve. With this formalism, they attacked a variety of problems, perhaps the most famous being superconductors in the presence of disorder. From their formalism, it became trivial to understand why non-magnetic impurities do not degrade superconductivity, whereas magnetic impurities do. Moreover, the latter degrade superconductivity in such a fashion that the energy gap, previously thought to be equivalent to the superconducting order parameter, actually vanishes before superconductivity disappears, leading to a novel region of gapless superconductivity. This body of work, as well as far more extensive work on "normal" materials, formed the basis of the famous textbook by the two of them and Igor Dzyaloshinskii, referred to in the introduction.

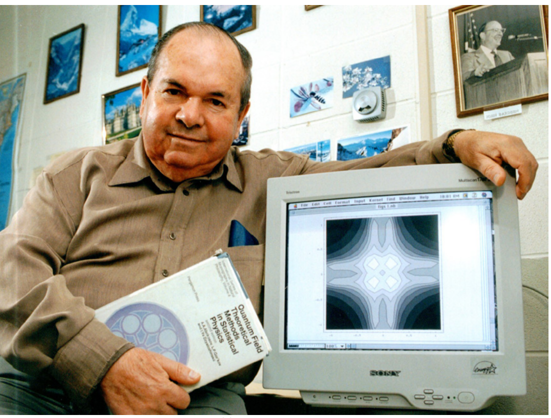
After this, Alex turned his attention to metals and semimetals. In 1965, he published a famous paper attacking the problem of magnetic impurities in a normal metal (the Kondo problem), again by summing selective (parquet) diagrams, where he ascertained the presence of a resonance (the Kondo, or Abrikosov-Suhl resonance, as Harry Suhl had independently derived the resonance the same year). A method Alex introduced was to express the spin operator as a Pauli matrix bracketed by so-called pseudo-fermions (now known as Abrikosov fermions). This was, to my knowledge, the first introduction of such auxiliary particles (commonly known as slave particles). Similar methods were later exploited by Piers Coleman and Nick Read in the 1980s to treat a lattice of Kondo impurities (the heavy fermion problem), and then in the 1990s by Phil Anderson, Patrick Lee, Nick Read, Subir Sachdev and Xiao-Gang Wen, among others, to address the problem of quantum spin liquids. These methods remain quite popular, particularly in the field of spin liquids, and led to Anderson's concept that spin ½ excitations could actually form a zero energy `spinon' Fermi surface, an idea going back to Pomeranchuk in 1941, and the existence of which is still being sought (several candidate materials have been proposed to have such surfaces).

It was also in the early 1960s that Alex first started working on semimetals (which are much in vogue these days with the discovery of topological varieties). With Leonid Falkovskii (with whom he had done an earlier paper on Raman scattering in superconductors), he considered the canonical example of bismuth (very recently discovered to be an ultra-low temperature superconductor), explaining the role of crystallographic deformation in yielding a semimetallic state (1962). In later work with Beneslavskii (1971), he considered the case of touching energy bands (with both linear and quadratic dispersion around the touching point), again of much relevance today with all of the excitement associated with Dirac and Weyl semimetals (the example they mentioned, HgTe, is quite prominent in recent literature). He also explored the possibility of an excitonic insulator phase in bismuth in a strong magnetic field (1973). For his work on semimetals and gapless superconductors, Alex would receive, along with some experimental colleagues, the State Prize of the USSR (1982). During the 1970s and early 1980s, he would also work on the electrical properties of quasi-one-dimensional metals (with Ryzhkin), including their superconducting properties.

During this time period, Alex moved around quite a bit. After Landau's car accident in 1962, the decision was made to found a theoretical institute in his name and move out of Moscow (from the Institute for Physical Problems that Alex had been at since 1948) to the village of Chernogolovka (a move which was obviously not relished by most). Alex, along with Isaak Khalatnikov, was instrumental in setting up this new institute. Although he was at the Landau Institute from 1965 to 1988, he also held positions in Moscow, first at the Moscow Institute of Physics and Technology (1972 to 1976), and then at the Moscow Institute of Steel and Alloys (1976 to 1991), and was also director of the Institute for High Pressure Physics from 1988 to 1991, at which time he departed for the United States. The lectures he gave in Moscow formed the basis of his classic textbook, *Fundaments of the Theory of Metals*, published in Russian in 1987, and in English the following year (a book that I again have put to great use over the years).

Two anecdotes from this period are perhaps worth sharing. The first was a story Alex related to me. During the Soviet era, foreign travel was particularly difficult to get permission for. Alex, being an avid mountaineer, along with some of his more enterprising colleagues, desired to climb Mount Kilimanjaro in Africa. But how to arrange it? They decided to call it a scientific expedition, and somehow got the authorities to sign off on it. Upon their return to Russia, the authorities wished to know what they had discovered during their expedition. After an embarrassing silence, one of the members suddenly spoke up: "It was cold." The other is related in Khalatnikov's memoirs. He and Alex were great champions of the practical joke. One took place during one of the symposia they held in Georgia. A shop that they kept passing had only a few items worth noting: framed portraits of Stalin (who was Georgian), long kitchen knives, and women's panties. The idea thus donned on them. They bought a knife and a pair of panties, and then hid them in Arkady Migdal's suitcase, along with a note reading, "you will answer for the outraged honor" appropriately translated to Georgian by Alex's graduate student, Rita (who was Georgian). They of course knew that Mrs. Migdal would be the one unpacking the bag when Arkady returned to Moscow, much to his embarrassment.

In the wake of perestroika, Alex decided that the future of science in Russia was not bright, and decided he would get away if he could. His initial thoughts were to go to France, given the presence there of his second son, but this prospect was not viewed well by some in France (Alex's second wife had been Philippe Nozieres' wife.) Word soon got out that Alex was looking around. Ivan Schuller at UCSD found out about this, and alerted Frank Fradin, who at that time was an associate laboratory director at Argonne. To throw off the authorities, Alex arranged a one month visit to Venezuela, and while he was on the visit, took a one week trip to Argonne to get acquainted with people there. When I first saw Alex during this visit, he reminded me of a farmer (he wore suspenders). But it was clear that he was an absolute treasure trove of information, and his presence would be of real benefit to the lab. So, Alex came in 1991, and six months later was appointed head of the Condensed Matter Theory Group, which he led from 1992 to 2000. This decision had its pluses and minuses. On the plus side, Alex was a great person to talk to, though you had to make sure you had plenty of time on your hands if you decided to venture into his office (he loved to tell stories). He also attracted many visitors, particularly Russians—Andrey Varlamov, Sasha Buzdin, Yuri Galperin, and Lev Pitaevskii were regular visitors, and Anatoli Larkin spent a six-month sabbatical (sadly, the Argonne administration moved too slowly and Anatoli left for Minnesota rather than staying at Argonne permanently.) One consequence of these visits was that Russian was commonly heard during seminars (sometimes from Alex himself, especially if he got bored with the speaker). One day, I arrived for a seminar late, only to be surprised to find out that the speaker, Sasha Finkelstein, was giving his lecture in Russian (his reaction was "ah, Mike is here,



Alex in his office at Argonne. Note the copy of AGD he is holding, and the picture of John Bardeen on the wall.

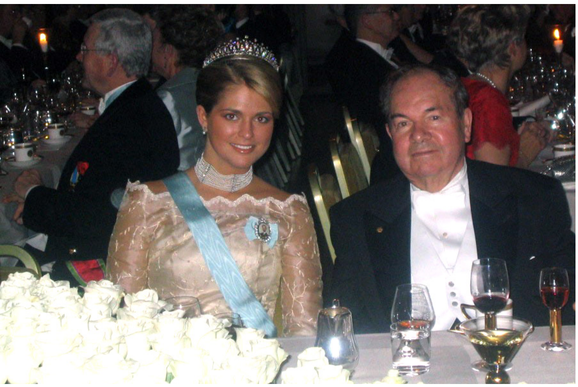
I must switch to English now.") This led to perhaps one of the more famous practical jokes during Alex's time at Argonne. Alex's postdoc, Tartakovskii, having seen the annoyance of some of the non-Russians to what was going on in the seminars, constructed an e-mail stating that it had come to Alex's attention that certain members of the theory group were deficient in the Russian language. Therefore, mandatory Russian language classes must be taken by these individuals. Now, there were two giveaways to this joke, first the date it was sent (April 1) and second the signature (Alex Apricot, "apricot" being the English translation of "abrikos"). Unfortunately, one of my colleagues did not notice these hints, and burst into my office saying that Alex had gone too far now, and he refused to take these language classes!

Although advanced in years, Alex did do some very interesting work while at Argonne. Perhaps his best-known work from this time was on quantum magnetoresistance (1998). A colleague at Argonne, Marie Louise Saboungi, had been providing samples of doped silver chalcogenides to Tom Rosenbaum's group at Chicago. A striking linear behavior with magnetic field was seen for the resistivity in these samples. Alex's explanation was drawn from his days working on semimetals, where he proposed that the material had a linear dispersion (nowadays, we would call this a Dirac semimetal), and with such a low carrier concentration, that with modest field one was already in the quantum limit where only one Landau level was relevant. Interest in his theory has since been renewed given all the recent excitement concerning topological semimetals (which this particular material has been conjectured to be).

Some of the participants from the symposium at Argonne in honor of Alex's 70th birthday (November 1998). From left to right, Helmut Brandt, Grisha Volovik, Dick Klemm, Boris Shklovskii, George Crabtree, Boris Altshuler, Lev Gor'kov, David Bishop, Alex, David Nelson, Mike Tinkham, Phil Anderson, Valerii Vinokur, Igor Dzyaloshinskii, and Dima Khmel'nitskii.

Alex pursued a number of other topics at Argonne. One was invoking van Hove singularities in the density of states as a mechanism to enhance superconductivity. As with most of his work, this project was motivated by experimental data, in this case that of Juan Carlos Campuzano's group (JC's office being next to Alex's), who found the presence of a flat band near the chemical potential in a cuprate superconductor. This flatness, Alex realized, leads to a power law divergence of the density of states (as opposed to the less singular logarithm associated with a normal van Hove singularity), and he derived the mean field consequences of such an electronic structure, particularly in regards to its influence on superconductivity. Alex also worked on how electronic transport occurs between layers (cuprates being a great example of weakly coupled planes), invoking the concept of resonant tunneling via defects to explain how the electrons moved from one layer to the next. Finally, based on some earlier work from 1978, he further pursued a potential mechanism of high Tc superconductivity where one replaces ions in the classic model for superconductivity by heavy holes in order to boost the energy scale.

In 2003, Alex won the Nobel Prize, giving him the opportunity to travel even more widely than before (a luxury he embraced given all the years he was denied travel while living in the Soviet Union). Unfortunately, six months later, he developed macular degeneration, completely losing all vision in one eye, and having very limited sight in the other. This began to limit both the travel and the work he could do. Alex, though, compensated for this. A way was found to blow up images on his computer monitor, and that, coupled with the jeweler glasses he wore, allowed him to do a certain amount of reading and writing. We offered him additional secretarial support (and a larger office), but Alex was too modest to accept such. Rather, he came in every day to the lab like clockwork (even after he officially retired in 2014), and for a long time even drove to the lab himself (we were rather terrified about this, and fortunately at some point, his wife, Svetlana, took over the driving duties)



Alex and Princess Madeleine at the Nobel Banquet (December 2003). I saw a picture of this scene in the Swedish papers the following day that cut Alex out!

Then, in the spring of 2016, Alex suffered a heart attack. His wife decided to move them to be with their daughter, Natalia, who was a physician living in Palo Alto. For a while, it seemed like he was doing better, but then he suffered another heart attack and died there on March 29, 2017.

Alex, married three times (Tatyana, Annie, and Svetlana, whom he was married to for forty years.) He had sons from his first two marriages (Alexey, a physicist as well, and Michel) along with a step daughter from his last (Natalia). Alex had a unique personality. He was opinionated, and shared more of his thoughts on people and events than he probably should have. He himself recognized this, which is the reason he never wrote his memoirs. Perhaps the most infamous example of this was an op-ed piece in the New York Times by Sergei Leskov from 1993, where Alex was quoted as saying that the best Russian scientists should be recruited away from Russia, and the rest should be ignored, which led to a howl of protests. On the other hand, he could be exceptionally kind. The reason I became a Fellow of the American Physical Society at such a young age was his nomination, among other kindnesses he bestowed on me over the years.

I will not dwell on the long list of prizes and honors Alex received during his long career (including induction into this Academy in 2000). Suffice it to say that the body of work that Alex left behind will be remembered for generations to come. I personally do not believe that AGD will ever go out of print. If you don't have a copy and you are a theoretical physicist, you should get one. There's no telling what gems you will find there.

ACKNOWLEDGEMENTS

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REFERENCES

Abrikosov, A. A. 1965. *Academician Lev Landau*. Originally in Russian, and recently translated into English (2015) by Alexey Galda, Ivan Sadovskyy, and Andrey Varlamov, with Michael Norman as scientific editor Argonne, IL: Argonne.

Andreev, A. F., et al. 2008. Aleksei Alekseevich Abrikosov (on his 80th birthday). *Physics Uspekhi* 51:637-638.

Andreev, A. F., et al. 2017. In memory of Aleksei Alekseevich Abrikosov. *Physics Uspekhi* 60:638-639.

Dokshitzer, Yu. L. 2004. QCD phenomenology. In *Proceedings of the 2002 European School of High-Energy Physics*. Pp 1-33. Geneva: CERN.

Geshkenbein, B. V. 2001. Introducing Boris Ioffe. In *At the Frontiers of Particle Physics: Handbook of QCD, Vol. 1*. Edited by M. Shifman. Pp. 9-14. Singapore: World Scientific.

Gor'kov, Lev. P. 2010. Developing BCS Ideas in the former Soviet Union. *Intl. J. Modern Phys. B* 24:3835-3854.

Hargittai, Balazs and István Hargittai. 2005. Alexei A. Abrikosov. in *Candid Science V: Conversations with Famous Scientists.* Pp 177-197. Singapore: World Scientific Publishing.

Khalatnikov, Isaak M. 2007. *From the Atomic Bomb to the Landau Institute.* Heidelberg: Springer.

Leskov, Sergei. 1993. America's Soviet Scientists (*New York Times*, July 15, 1993).

Polyakov, A. 1992. A view from the island. https://arxiv.org/abs/hep-th/9211140

Varlamov, Andrey. 2017. Alex Abrikosov: life and scientific biography. XXIV Davydov Lectures (Kiev, Ukraine, Dec. 26, 2017).

Wikipedia. 2017. Alexei Alexeyevich Abrikosov, https://en.wikipedia.org/wiki/Alexei_Alexeyevich_Abrikosov

SELECTED BIBLIOGRAPHY

1952 The influence of size on the critical field of the superconductors of the second group. *Doklady Akademii Nauk SSSR* 86:489-492.

1954 On the internal structure of hydrogen planets. *Voprosy Kosmogonii* 3:11-19.

With I. M. Khalatnikov and L. D. Landau. Asymptotics of the photon Green function in quantum electrodynamics. *Doklady Akademii Nauk SSSR* 95:1177-1180.

1956 On the infrared catastrophe in quantum electrodynamics. *Soviet Phys.* 3:71-80.

With I. M. Khalatnikov and L. D. Landau. On the quantum theory of fields. *Il Nuovo Cimento* 3 (Suppl. 1):80-104.

1957 On the magnetic properties of superconductors of the second group. *Soviet Phys. JETP* 5:1174-1182.

1959 With I. M. Khalatnikov. The theory of a Fermi liquid (the properties of liquid ^{3}He at low temperatures). *Rep. Prog. Phys*. 22:329-367.

With L. P. Gor'kov. On the theory of superconducting alloys I. The electrodynamics of alloys at absolute zero. *Soviet Phys. JETP* 35:1090-1098.

With L. P. Gor'kov and I. M. Khalatnikov. A superconductor in a high frequency field. *Soviet Phys. JETP* 35:182-189.

With L. P. Gor'kov and I. E. Dzyaloshinskii. On the application of quantum-field-theory methods to problems of quantum statistics at finite temperatures. *Soviet Phys. JETP* 36:636-641.

1961 With L. P. Gor'kov. Contribution to the theory of superconducting alloys with paramagnetic impurities, *Soviet Phys. JETP* 12:1243-1253.

With L. A. Fal'kovskii. Raman scattering of light in superconductors. *Soviet Phys. JETP* 13:179-184.

1962 With L. P. Gor'kov. Spin-orbit interaction and the Knight shift in superconductors, *Soviet Phys. JETP* 15:752-757.

With L. P. Gor'kov and I. E. Dzyaloshinskii. *Methods of Quantum Field Theory in Statistical Physics*. Englewood Cliffs, N.J.: Prentice-Hall. (Revised edition, New York: Dover, 1975.).

1963 With L. A. Fal'kovskii. Theory of the electron energy spectrum of metals with a bismuth type lattice. *Soviet. Phys. JETP* 16:769-777.

1965 Electron scattering on magnetic impurities in metals and anomalous resistance effects. *Physics* 2:5-20.

1970 With A. A. Migdal. On the theory of the Kondo effect. *J. Low. Temp. Phys.* 3:519-536.

1971 With S. D. Beneslavskii. Possible existence of substances intermediate between metals and dielectrics. *Soviet Phys. JETP* 32:699-708.

1973 Transition of a bismuth-type semimetal to an excitonic insulator in a strong magnetic field. *J. Low Temp. Phys.* 10: 3-34.

1978 With I. A. Ryzhkin. Conductivity of quasi-one-dimensional metal systems. *Adv. Phys.* 27:147-230.

Possible mechanism of high-temperature superconductivity. *JETP Lett.* 27:219-222.

1987 *Fundamentals of the Theory of Metals.* New York: North Holland. (Reprint, New York: Dover, 2017).

1994 With K. Gofron, et al. Observation of an "extended" van Hove singularity in $YBa_2Cu_4O_8$ by ultrahigh energy resolution angle-resolved photoemission, *Phys. Rev. Lett.* 73:3302-3305.

1998 Quantum magnetoresistance. *Phys. Rev. B* 58: 2788-2794.

2004 Nobel Lecture: Type-II superconductors and the vortex lattice. *Rev. Mod. Phys*. 76:975-979.